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What Multi-Level Modeling Can Teach Us About Single-Level Modeling & Vice Versa: The Case of Latent Transition Analysis

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Mplus: `www.statmodel.com`

PSMG Presentation April 14, 2020

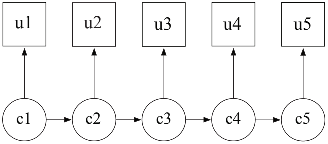
I thank Tihomir Asparouhov for helpful comments and Noah Hastings for excellent assistance

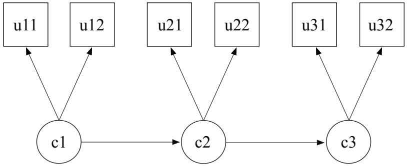
Outline

- Modeling:
  - Statistics
  - Psychology
  - Psychometrics
- Applications:
  - Mood data (Eid & Langeheine, 2003)
  - Reading proficiency (Kaplan, 2008)
  - Dating and sexual risk behavior data (Lanza & Collins, 2008)
- Multi-level time series analysis (DSEM)
- Appendix A: Sample size requirements
- Appendix B: Computing time
- Appendix C: New features in Mplus

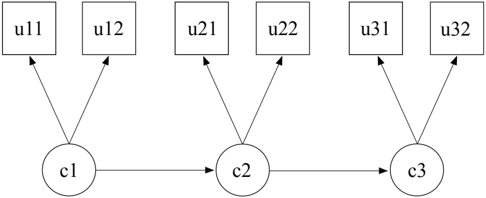
Hidden Markov - Latent Transition Analysis

u: observed categorical variable (latent class indicator)
c: latent categorical variable (latent class variable)





Lanza & Collins (2008). A new SAS procedure for latent transition analysis: Transitions in dating and sexual risk behavior. Developmental Psychology, 44, 446-456.

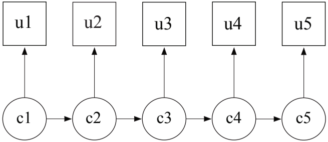


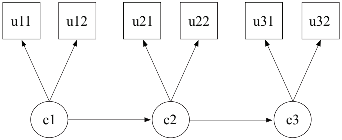
1. Initial status probabilities: $P(C_1)$
2. Transition probabilities: $P(C_2|C_1)$, $P(C_3|C_2)$
3. Measurement probabilities: $P(U_t|C_t)$ - LCA for each time point, measurement invariance across time

Extensions:

- Mover-Stayer modeling
- Multiple-group analysis: Measurement invariance
- Covariates: Influencing latent class probabilities and transition probabilities

What’s Missing in these Models?





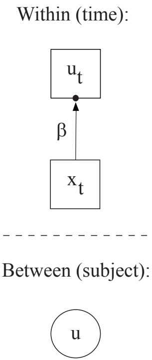
- Single indicator per time point
  - **A statistical perspective**
    - Multilevel modeling: Level 1 = time, level 2 = subject
    - Random effects, especially random intercepts
  - A substantive perspective
    - Trait theory in psychology
    - Between-subject differences that are stable over time
- Multiple indicators per time point
  - A psychometric perspective
    - Multilevel factor analysis
    - Multilevel latent class analysis

Two representations of regression of a binary U on a covariate X at time t for subject j using a random intercept u_j:

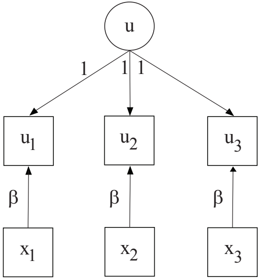
$$Logit\,[P(U_{tj}=1|X_{tj})] = u_j + \beta\, X_{tj}, \tag{1}$$

$$u_j = u + \varepsilon_j.\;\; V(\varepsilon_j) = \sigma^2_u \tag{2}$$

Long format, 2-level:



Wide format, single-level:

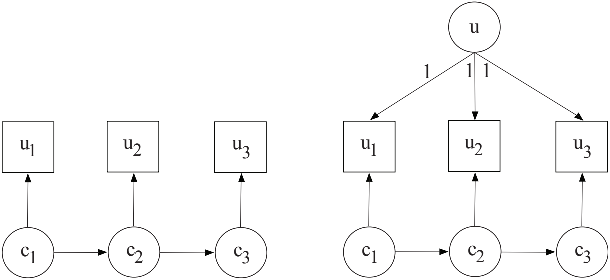


An Aside: Panel Data, Time-Invariant Omitted Variables, Fixed vs Random Effects

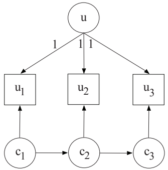
$$Logit\,[P(U_{tj} = 1|X_{tj})] = u_j + \beta\, X_{tj} = u + \varepsilon_j + \beta\, X_{tj}. \qquad (3)$$

- Allison (2005). Fixed effects regression methods for longitudinal data using SAS. Cary, NC: SAS Instiute Inc.
- Allison et al. (2017). Maximum likelihood for cross-lagged panel models with fixed effects. Sociological Research for a Dynamic World
- Wooldridge, J. M. (2013). Introductory econometrics: A modern approach (5th ed.). Mason, OH: South-Western, Cengage Learning
- Bell & Jones (2015). Explaining fixed effects: Random effect modeling of time-series cross-sectional and panel data. Political Science Research and Methods, 3 , 133-153
- Hamaker & Muthén (2019). The fixed versus random effects debate and how it relates to centering in multilevel modeling. Forthcoming in Psychological Methods

Single-Indicator LTA (Hidden Markov)



- The Hidden Markov model on the left can use the fast backward-forward Baum-Welch algorithm for ML estimation
- The random intercept model on the right loses this simplicity - the U's are no longer independent conditional on the Cs
  - ML estimation requires numerical integration with much heavier computations



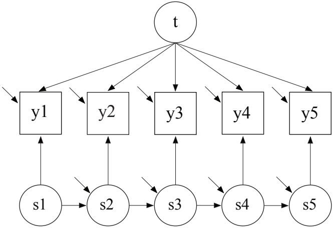
Altman (2007). Mixed hidden Markov models. Journal of the American Statistical Association, 102, 201-210.

- Relapsing-remitting multiple sclerosis patients, N=39
  - Symptoms worsen and then improve in alternating periods of relapse and remission
- Outcome: Number of lesions in the brain (count variable)
- T=24 (monthly scans for two years)
- 2 latent classes (hidden states): Relapse versus remission

What's Missing?

- Single indicator per time point
  - A statistical perspective
    - Multilevel modeling: Level 1 = time, level 2 = subject
    - Random effects, especially random intercepts
  - **A substantive perspective**
    - Trait theory in psychology
    - Between-subject differences that are stable over time
- Multiple indicators per time point
  - A psychometric perspective
    - Multilevel factor analysis
    - Multilevel latent class analysis

Kenny & Zautra (1995). The trait-state-error model for multiwave data. Journal of Consulting and Clinical Psychology.

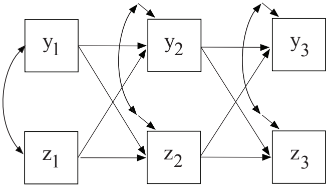


- t represents a continuous latent trait variable
- y1-y5 represent continuous observed variables
- s1-s5 represent continuous latent state variables

Latent Trait-State Analysis Continued

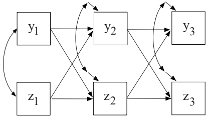
- Cole et al. (2005). Empirical and conceptual problems with longitudinal trait-state models: Introducing a trait-state-occasion model. Psychological Methods
- Kenny & Zautra (2001). Trait-state models for longitudinal data. In A. Sayer & L. M. Collins (Eds.), New methods for the analysis of change (pp. 243-263). Washington, DC: American Psychological Association. - STARTS model
- Wagner at al. (2015). Self-esteem is mostly stable across young adulthood: Evidence from latent STARTS models. Journal of Personality
- Ludtke et al. (2018). More stable estimation of the STARTS model: A Bayesian approach using Markov Chain Monte Carlo techniques. Psychological Methods

Cross-Lagged Panel Modeling (CLPM)



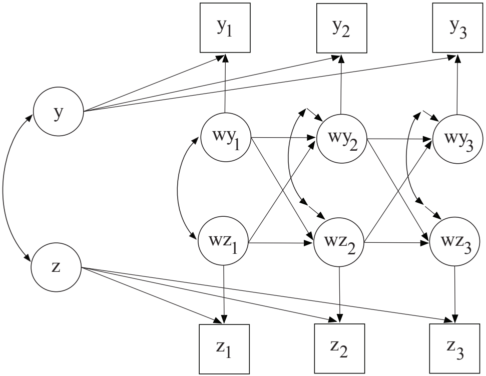
Hamaker, Kuiper, Grasman (2015). A critique of the cross-lagged panel model. Psychological Methods, 1, 102-116.

Hamaker et al. (2015) Critique of CLPM



- ”if stability of constructs is to some extent of a trait-like, timeinvariant nature, the autoregressive relationships of the CLPM fail to adequately account for this”
- ”As a result, the lagged parameters that are obtained with the CLPM do not represent the actual within-person relationships over time”
- ”this may lead to erroneous conclusions regarding the presence, predominance, and sign of causal influences
- an alternative model ... **separates the within-person process from stable between-person differences through the inclusion of random intercepts**”

Random Intercept Cross-Lagged Panel Model (RI-CLPM)



Hamaker et al. (2015)

Mplus scripts:

`http://www.statmodel.com/RI-CLPM.shtml`

What's Missing? Multiple-Indicator Case

- Single indicator per time point
  - A statistical perspective
    - Multilevel modeling: Level 1 = time, level 2 = subject
    - Random effects, especially random intercepts
  - A substantive perspective
    - Trait theory in psychology
    - Between-subject differences that are stable over time
- **Multiple indicators per time point**
  - **A psychometric perspective**
    - Multilevel factor analysis
    - Multilevel latent class analysis

Multiple Indicators in Cross-Sectional Data: Multilevel Factor Analysis with Random Intercepts

- Binary factor indicators U_{ij} for subject i in cluster j
- Typical example: measurement of student performance in schools
- $f_{W_{ij}}$ and f_{B_j} represent within- and between-level variation in the factor

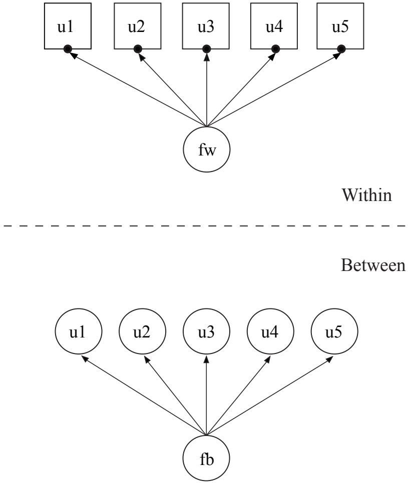
For each U_{ij} factor indicator, the model can be expressed by the two equations (level-1 and level-2):

$$logit[P(U_{ij} = 1 | f_{W_{ij}})] = u_j + \lambda_W f_{W_{ij}}, \tag{4}$$

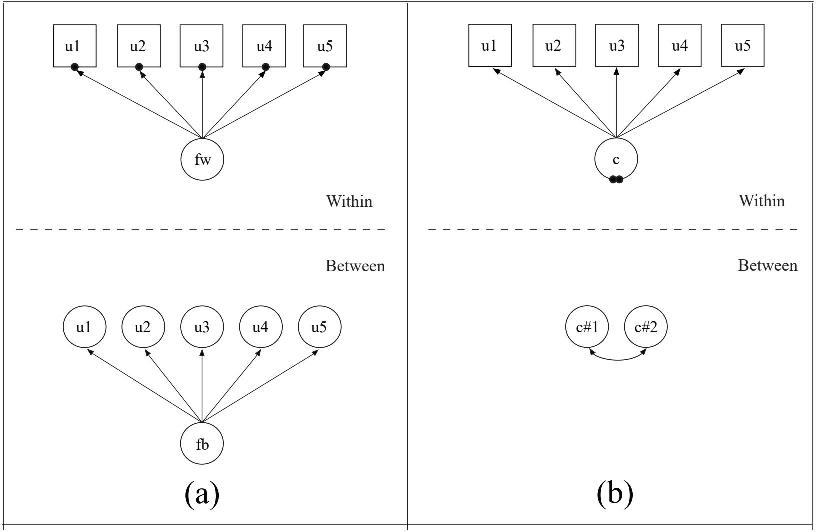
$$u_j = u + \lambda_B f_{B_j} + \varepsilon_{Bj}, \tag{5}$$

This is in line with two-level regression where the intercept u_j is random, varying across schools. The ε_{Bj} residuals are typically small.

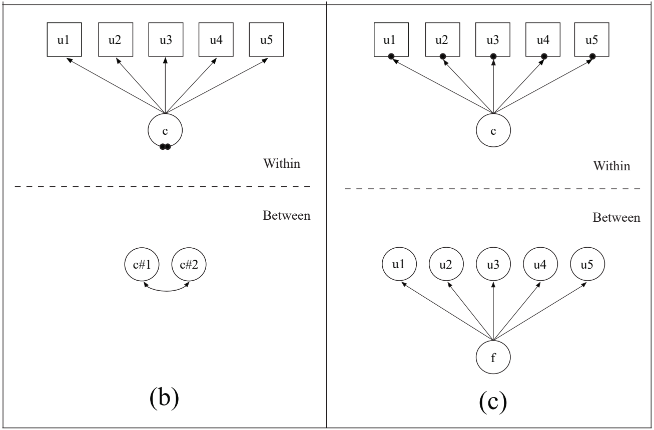
Multilevel Factor Analysis with Random Intercepts



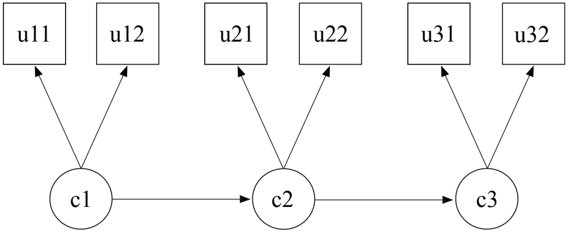
Multilevel Factor Analysis versus Latent Class Analysis



Vermunt (2003, 2008), Asparouhov-Muthén (2008), Henry-Muthén (2010)



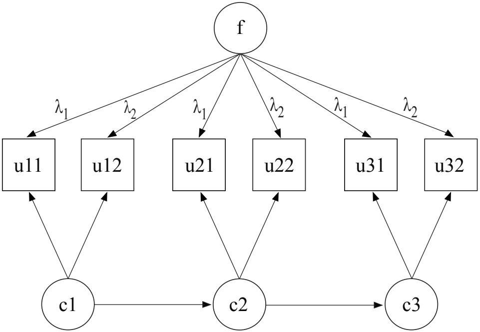
Back to Multiple-Indicator Latent Transition Analysis



1. Initial status probabilities: $P(C_1)$
2. Transition probabilities: $P(C_2|C_1)$, $P(C_3|C_2)$
3. Measurement probabilities: $P(U_t|C_t)$ - LCA for each time point, measurement invariance across time

- What's missing?

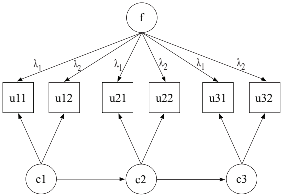
Muthén & Asparouhov (2020). Random intercept latent transition analysis (RI-LTA). Under review.



Mplus scripts:

`http://www.statmodel.com/RI-LTA.shtml`

Features of RI-LTA



- Fits the data much better than regular LTA which is unnecessarily restrictive and gives distorted results
- Does not confound between- and within-subject sources of variation: $C_{t-1} -> C_t$ represents a within-subject process free of time-invariant between-subject differences (trait differences)
- Latent class indicators correlate over time beyond what is captured by the latent class correlations over time:
  - Tends to reduce the probability of subjects staying in the same class as compared to regular LTA
  - Reduces the need for Mover-Stayer modeling because Movers and Stayers can be captured by different random intercept values

Software Improvements for LTA and RI-LTA

RI-LTA can be time consuming due to numerical integration and needing many random starts to find the global maximum.
Mplus Version 8.4 released last November:

- Significant speed improvements for computationally demanding mixture models such as with LTA and RI-LTA using a new three-stage random starts search and using specialized algorithms drawing on Baum-Welch ideas
- Asparouhov & Muthén (2019). Random Starting Values and Multistage Optimization (Technical Report: `http://www.statmodel.com/download/StartsUpdate.pdf`)
  - "A 20 hours computation in Mplus 8.3 can be done in Mplus 8.4 in less than 15 minutes, by utilizing the advantages of the three-stage estimation, the Baum-Welch algorithm, as well as updated hardware (i9-9900k Intel CPU)"
- Substantially simplified output for mixture models with multiple latent class variables

Outline

- Modeling:
  - Statistics
  - Psychology
  - Psychometrics
- **Applications:**
  - Mood data (Eid & Langeheine, 2003)
  - Reading proficiency (Kaplan, 2008)
  - Dating and sexual risk behavior data (Lanza & Collins, 2008)
- Multi-level time series analysis (DSEM)
- Appendix A: Sample size requirements
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Regular LTA Fits Worse than RI-LTA Most of the Time

Analyses of 4 data sets from the LTA literature (data at `http://www.statmodel.com/RI-LTA.shtml`):

Life Satisfaction (non-stationary)
N=5147, T=5, R=1, J=5

Mood (stationary)
N=494, T=4, R=2, J=2

Model	# par's	logL	BIC	Model	# par's	logL	BIC
Regular LTA	11	-15326	30745	Regular LTA	7	-2053	4150
RI-LTA	12	-15267	30637	RI-LTA	9	-2018	4093

Reading proficiency (non-stationary)
N=3574, T=4, R=5, J=3

Dating and sexual risk behavior (stationary)
N=2933, T=3, R=3, J=5

Model	# par's	logL	BIC	Model	# par's	logL	BIC
Regular LTA	35	-21793	43873	Regular LTA	49	-16202	32796
RI-LTA	40	-20329	40984	RI-LTA	52	-16043	32502

Mood data. Eid & Langeheine (2003)

- German Multidimensional Mood Questionnaire data, N=494, ages 17-77 (Steyer et al., 1997)
- 4 time points 3 weeks apart
- 2 binary items based on ratings of momentary sadness and unhappiness
- Stationary model (equal transition probabilities over time)
- Eid & Langeheine (2003). Separating stable from variable individuals in longitudinal studies by mixture distribution models. Measurement: Interdisciplinary Research and Perspectives

Model Fit for Mood Data

Model	# parameters	log likelihood	BIC
		Standard	
1 Regular LTA	7	-2053	4150
2 RI-LTA	9	-2018	4093

Mood Data Measurement Probability Estimates

	Classes	
Regular LTA		
Indicator	Not sad/Happy	Sad/Unhappy
Sad	0.089	0.902
Unhappy	0.038	0.857
RI-LTA		
Indicator	Not sad/Happy	Sad/Unhappy
Sad	0.286	0.748
Unhappy	0.164	0.804

Mood Data Transition Probability Estimates

2 latent classes: Not Sad/Happy and Sad/Unhappy

	Classes	
Regular LTA		
Classes	Not sad/Happy	Sad/Unhappy
Not sad/Happy	0.803	0.197
Sad/Unhappy	0.248	0.752
RI-LTA		
Classes	Not sad/Happy	Sad/Unhappy
Not sad/Happy	0.691	0.309
Sad/Unhappy	0.486	0.514

Model Fit for Mood Data: Mover-Stayer Modeling

Model	# parameters	log likelihood	BIC
Standard			
1 Regular LTA	7	-2053	4150
2 RI-LTA	9	-2018	4093
Mover-Stayer			
3 Regular LTA	8	-2037	4123
4 RI-LTA	10	-2017	4096

- Unlike regular LTA, RI-LTA does not need a Mover-Stayer component
- Large negative and positive intercept factor scores capture Stayers

Reading Proficiency. Kaplan (2008)

- Early Childhood Longitudinal Study (ECLS), N = 3574
- 4 time points: Fall and Spring of Kindergarten and Fall and Spring of Grade 1
- 5 binary items representing a stage-sequential process:
  - Basic reading skills of letter recognition
  - Beginning sounds
  - Ending letter sounds
  - Sight words
  - Words in context
- 3 latent classes corresponding to 3 stages of learning
- Kaplan (2008). An overview of Markov chain methods for the study of stage-sequential developmental processes. Developmental Psychology

Model Fit for Reading Data

Model	# parameters	log likelihood	BIC
	Standard		
Regular LTA	35	-21793	43873
RI-LTA	40	-20329	40984

Reading Data Measurement Probability Estimates

Class 1 = low alphabet knowledge, Class 2 = early word reading, Class 3 = early reading comprehension

	Regular LTA			RI-LTA		
	Classes			Classes		
	1	2	3	1	2	3
Letrec	0.505	0.994	1.000	0.627	0.939	0.979
Begin	0.066	0.917	0.984	0.303	0.806	0.941
Ending	0.013	0.661	0.972	0.167	0.630	0.904
Sight	0.000	0.051	0.985	0.020	0.208	0.808
WIC	0.000	0.001	0.509	0.005	0.058	0.460

Reading Data Transition Probability Estimates

	Regular LTA			RI-LTA		
Latent	Fall 1st -– Spring 1st			Fall 1st -– Spring 1st		
Classes	1	2	3	1	2	3
1	**0.263**	0.505	0.232	**0.154**	0.000	0.845
2	0.005	**0.132**	0.863	0.004	**0.019**	0.977
3	0.001	0.000	**0.999**	0.009	0.000	**0.991**

The random intercept factor f can perhaps be viewed as a reading preparedness dimension. It is strongly related to poverty

Transitions in Dating and Sexual Risk Behavior
Lanza and Collins (2008) Developmental Psychology

- Data:
  - National Longitudinal Survey of Youth (NLSY97), N=2937
  - 3 time points one year apart starting at ages 17-18
- Items:
  - Past-year number of dating partners (0, 1, 2 or more)
  - Past-year number of sexual partners (0, 1, 2 or more)
  - Exposed to STD in past year (no, yes)

Model Results for Dating Data

- Regular LTA (# pars = 49, logL = -16202, BIC = 32796):
  - 5 classes: Nondaters, Daters, Monogamous, Multipartner-Safe, Multipartner-Exposed
- RI-LTA (# pars = 52, logL = -16043, BIC = 32502):
  - 5 classes but only Nondaters, Daters, and Multipartner-Exposed correspond to the solution obtained with regular LTA

Model Fit for Dating Data
Adding a Mover-Stayer Component

Model	# par's	logL	BIC
Standard			
Regular LTA	49	-16202	32796
RI-LTA	52	-16043	32502
Mover-Stayer			
Regular LTA	50	-16194	32787
RI-LTA	53	-16041	32506

Dating Data Estimates

RI-LTA allows a continuum of Staying instead of a Stayer class:

- RI-LTA model captures staying also via different intercept factor values
- RI-LTA does not need a Stayer class or the Stayer class is smaller
  - Regular LTA Mover-Stayer model estimates 20% Stayers
  - RI-LTA Mover-Stayer model estimates 9% Stayers

RI-LTA makes it possible to learn more about the indicators:

- Estimated RI-LTA intercept factor loadings (all significant):
  - # dating partners = 1.55
  - # sex partners = 4.22 (strongest indicator of the random intercept factor, that is, the time-invariant trait)
  - exposed to STD = 1.51

RI-LTA Model Variations

- Lag-2, lag-3 etc for the latent class variables
- Distal outcomes
- Time trend
- Multiple processes (cross-lagged RI-LTA)
- Multilevel RI-LTA

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    - Statistics
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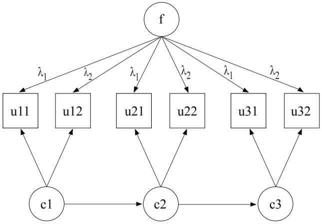
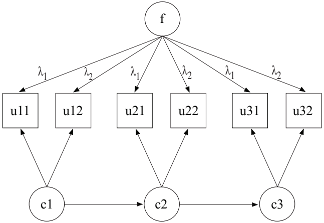
The Vice Versa: What Single-Level Modeling Can Teach Us About Multi-Level Modeling



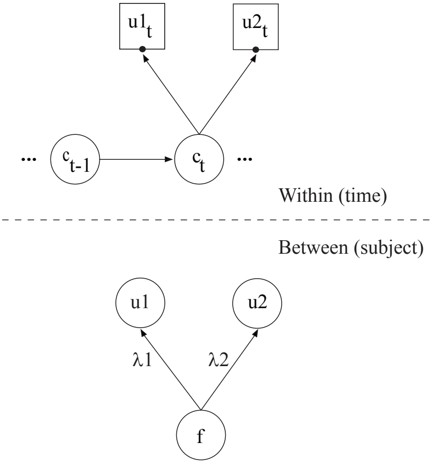
- How do you do analyze relations between variables at different time points using two-level modeling?
  - Multilevel time series analysis (time and individual)
  - Other random effects can be added: Random slopes, random variances, random AR, random transition probabilities
  - Many time points required - intensive longitudinal data (EMA, ESM, daily diary)

Dynamic Structural Equation Modeling (DSEM)

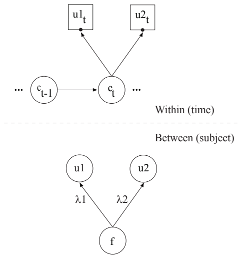
Single-level, wide



Two-level, long, time series DSEM



- Dynamic structural equation modeling (DSEM): Asparouhov et al. (2017, 2018). Papers at `http://www.statmodel.com/TimeSeries.shtml`



- Two-level analysis: random effects varying across subjects
- Cross-classified analysis (of time and subject): random effects varying across subjects and time
- Data in long format:
  - 2 outcomes per time point results in 2 columns of data (not 2*T)
  - Across-time effects specified by using lags: C ON C&1 (lag 1)

Appendix A: Sample Size Requirements

- LTA typically needs large samples ($> 1{,}000$)
- RI-LTA (limited Monte Carlo simulation study in Muthén-Asparouhov, 2020; binary latent class indicators):
  - $T \geq 3$: Good performance at $N \geq 1{,}000$
  - $T = 2$: $N > 4{,}000$ needed
    - RI-LTA needs larger samples than regular LTA (assuming each model is correct)
    - But if data have been generated by RI-LTA, RI-LTA estimates are better than regular LTA at sample size as low as N=500
- DSEM (continuous outcomes):
  - Schultzberg-Muthén (2018). Number of subjects and time points needed for multilevel time series analysis: A simulation study of dynamic structural equation modeling. SEM journal
  - $T \geq 10$ for minimal models, otherwise $T \geq$ 20-50

Appendix B: RI-LTA Computing time

Timings using different Intel CPUs (hours:minutes:seconds)

	i7-7700k Reg LTA	i7-7700k RI-LTA	i9-9900k RI-LTA Proc=8	i9-9900k RI-LTA Proc=12
Mood (N=494, T=4, 2 classes)	00:00:04	00:00:12	-	-
Dating (N=2937, T=3, 5 classes)	00:00:41	00:06:00	00:03:31	00:03:25
Dating (N=2937, T=3, 5 classes, adding 4 covariates)	00:05:00	00:40:00	00:21:00	00:19:00

Appendix C: New Mplus Features in Version 8.3 and Version 8.4

http://www.statmodel.com/verhistory.shtml

- Mixture modeling:
  - Significant speed improvements for computationally demanding mixture models, particularly with multiple latent class variables such as with Latent Transition Analysis
  - Substantially simplified output for mixture models with multiple latent class variables
- Bayesian analysis:
  - Significant speed improvements for Bayesian computations using a new parallelized computing approach (use PROCESSORS = 8)
  - Bayesian estimation of two-level models with latent variable interactions using the XWITH option. This is especially helpful for models with moderation where ML has problems
  - Expanded Bayesian fit statistics in 3 areas: improved posterior predictive p-values (PPP) when there are missing data, Bayesian CFI/TLI/RMSEA including confidence intervals, and a Bayesian version of the Wald test of parameter restrictions using the MODEL TEST command

Some Recent Mplus-Related Papers

- Asparouhov & Muthén (2019). Random Starting Values and Multistage Optimization (Technical Report)
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- Asparouhov & Muthén (2019). Bayesian estimation of single and multilevel models with latent variable interactions (under review)
- Asparouhov & Muthén (2019). Advances in Bayesian Model Fit Evaluation for Structural Equation Models (under review)
- Asparouhov & Muthén (2019). Latent variable centering of predictors and mediators in multilevel and time-series models. Structural Equation Modeling: A Multidisciplinary Journal, 26, 119-142.
- Asparouhov & Muthén (2020). Comparison of models for the analysis of intensive longitudinal data. Structural Equation Modeling: A Multidisciplinary Journal, 27(2) 275-297
- Muthén & Asparouhov (2020). Random intercept latent transition analysis (RI-LTA) (under review)